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A MODEL FOR STRUCTURAL CHANGES OF BELIEF

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ABSTRACT

The paper suggests a way of modeling belief changes within the tradition of formal belief revision theories. The present model extends the scope of traditional proposals, such as AGM, so as to take care of “structural belief changes” – a type of radical shift that is best illustrated with, but not limited to, instances of scientific discovery; we obtain AGM expansions and contractions as limiting cases. The representation strategy relies on a non-standard use of a semantic machinery. More precisely, the model seeks to correlate knowledge states with interpretations of a given formal language L , in such a way that the epistemic state of an agent at a given time gives rise to a picture of how things could be, if there weren't anything else to know. Interpretations of L proceed along supervaluational ideas; hence, the model as a whole can be seen as a particular application of supervaluational semantics to epistemic matters.

Keywords: Belief revision, Supervaluations, AGM-theory, Radical belief changes

§ 1. Introduction

Formal theories of belief revision aim at modeling the structure of changes that agents may undergo in their epistemic states. They are concerned with the formal aspects of a dynamics of beliefs, and not with the material content of the changes. Typically, they propose, in the first place, a way of representing the current epistemic state of a certain agent; the agent is frequently taken to be a human being, though this is not necessary (computers or institutions can do as well). In the second place, they suggest strategies for modeling the process of acquiring new beliefs or getting rid of old ones. With few exceptions, they do not intend to be *psychological* descriptions. Rather, the whole enterprise is conceived in a normative way (much in the way logic is normative and not descriptive): the goal is to capture some essential features that any dynamics of beliefs should exhibit, in order for the agent to be considered an ideally rational being. Thus, to build up a theory of belief revision amounts at the same time to a way of shaping some basic elements of a certain conception of rationality.

The aim of this paper is to offer, within the tradition of formal belief revision theories, a modeling strategy that could take care of *radical* shifts of belief, in a sense of radicality to be explained soon. The representation strategy will rely on a non standard use of a semantic machinery, according to which we will be able to correlate knowledge states with interpretations of a given formal language L . Interpretations of L will proceed along supervaluational ideas; hence, the model as a whole could be seen as a particular application of supervaluational semantics to epistemic matters.

Some of the details of the formal machinery have been picked out so as to obtain AGM-style of changes as limiting cases – where *AGM theory* is the proposal developed by Alchourrón, Gärdenfors and Makinson in the eighties.¹ The reason for this choice is not so much that I think AGM theory constitutes the best option in the market for ordinary (*i.e.*, non radical) epistemic changes, but that it is, in many respects, the standard point of departure at the time of discussing these issues. Notwithstanding, the core idea of

¹ Alchourrón *et al.* (1985); Gärdenfors (1988).

the present account (that is to say, the use of a supervaluational semantics of sorts to account for several types of epistemic shifts, including *radical* shifts) could be applied in a somewhat different fashion to yield as limiting cases other proposals rival to AGM.

§ 1.1. Motivating Structural Changes

Very briefly, AGM's main features are as follows:

- Agents are assumed to be able to accept, reject, or suspend judgment on sentences of a suitably regimented propositional language L .
- The epistemic state of the agent is represented as a “belief set” K of sentences of L – the sentences the agent “accepts.” K is consistent and deductively closed.
- Changes of belief are interpreted as functions from pairs consisting of a belief set and a sentence, to belief sets.
- The addition of a sentence A to K (a new “belief” of the agent), when A is consistent with K , is called *expansion*. It is represented as $K^+_A = Cn(K \cup \{A\})$, where Cn is the operator for logical consequence – *i.e.*, $Cn(K \cup \{A\})$ is the set of all logical consequences of $K \cup \{A\}$.
- The removal of a sentence A from K (meaning that the agent no longer accepts A) is called *contraction*, and represented as K^-_A .
- A *revision* K^*_A takes place when K is enlarged with a sentence A that may be inconsistent with K , in which case some elements in K have to be given up in order to restore consistency.
- AGM recipe for contraction and revision is the so-called *partial meet contraction and revision* functions. Thus, for the case of *contractions*, $K^-_A = \cap \gamma(K \perp A)$, where $K \perp A$ is the set of all belief sets K' that are maximal subsets of K that fail to imply A , and γ is a selection function that picks out a non empty subset of $K \perp A$ whenever $K \perp A$ is not empty (otherwise $\gamma(K \perp A) = K$). *Revisions* can be obtained with the aid of the so-called “Levi identity”: $K^*_A = (K^-_{\neg A})^+_A$.

As is well known, AGM theory, as well as most of its variants, only deals with

minimal changes.² But sometimes it is rational for agents to change their minds in ways that are far from minimal. Consider, for example, a reasonably competent speaker of everyday English – call it Tom – with no scientific education at all. Suppose also that, at some point, our agent begins to study astronomy, and that, at time t_1 , he ends up accepting the statement “there is a black hole in the active galactic nucleus of galaxy g .” Clearly, Tom’s epistemic state has been modified. But what type of change was that, exactly? Regardless of how we characterize it, it should be clear that it was not an AGM-style expansion. Had it been an AGM expansion, we would be forced to say that at time t_0 Tom suspended judgment about the statement. But, intuitively, Tom was not *in suspense* about it at t_0 – at least not in the same sense in which he might have been ignorant of, say, whether his friend’s birthday was or was not in May. In the former case, Tom did not have an attitude towards the statement at all: by hypothesis, he lacked the very concepts of “black hole,” or “active galactic nucleus”; there were no galaxies in his ontology, and “ g ” named nothing for him. Faced with Tom’s example, all AGM can do is claim that the idea under consideration *was not entertainable* for Tom at t_0 , and hence that it should not have been represented by any sentence of L at t_0 ; however, AGM-style theories do not offer any precise instructions as to how L could be allowed to change.

The stated distinction between being in suspense about a statement and being unaware of its very possibility should not be confused with the contrast between explicit or “occurrent” propositional attitudes and implicit ones. Indeed, it may well be irrelevant whether an agent has consciously assessed a particular idea, insofar as it remains, say, as a component part of the agent’s standing disposition to act, or perhaps insofar as it is a logical consequence of other commitments of the agent – depending on our preferred theory of beliefs. Assuming Tom has never thought about his friend’s birthday explicitly, we can still concede that he has a “non-occurrent” suspension of judgment. But nothing of the sort can be conceded regarding the astronomy case. It is not only that he has never

² For an interesting discussion of the “dogma” of minimal change see Rott (2000). For a recent work aimed at showing that AGM is not minimal enough, see Tennant (2006).

considered the issue explicitly. Rather, the point is that, had somebody raised the problem to him, he would have found it altogether meaningless.³

I shall dub the type of change illustrated above *propositional structural change*, or, for short, *structural change*. Let me say at the outset that I do not intend to identify structural shifts with *revolutionary* changes, in any loose Kuhnian sense (from Kuhn 1962). Granted, *some* examples of structural changes may also fit with (at least some of) Kuhn's criteria, but I shall not require so. In addition, "structural change" should not be read as a proxy for "*conceptual change*" (or "*conceptual innovation*"). Whether there is any overlapping between the meanings of the two expressions depends to a large extent on our prior understanding of concepts; it is worth noticing that many popular theories of conceptual change just do not capture the full generality of the phenomenon that I am trying to describe here.⁴

In what follows, the expression "structural *strengthening*" will refer to the shift that occurs when an agent comes to acknowledge the very possibility of formulating a novel hypothesis. In this case, the agent focuses her attention on a statement that has never occurred to her before, and to which she has never been exposed in the past – say, because it has never been entertained in her community before (as in cases of scientific discoveries) or because of particular biographical circumstances (as in our earlier example about Tom). Thus, in a structural strengthening the agent suddenly becomes aware of the very possibility of accepting, rejecting, or suspending judgment about a

³ To put it differently, agents credited with a *bona fide* suspension of judgment concerning *p* (even if it is a non-occurrent suspension of judgment) typically answer, "I don't know," when forced to consider whether *p* is the case (at which point they make the suspension of judgment explicit, of course); agents with no doxastic attitude towards *p*, by contrast, tend to react with perplexity – they might ask, for instance, "what are you talking about?" or "what do you mean?" rather than say, "I don't know." Naturally, in practice some examples might be hard to classify, but as long as paradigmatic cases are clear enough, vagueness – related problems should not count against the distinction. (Thanks to an anonymous referee for pressing this issue).

⁴ The main reason is that, as we shall see, the enlargement of a given propositional structure (what I shall call a "structural strengthening") need not be accompanied by a standard belief expansion. Thus, there will be cases in which the agent actually acquires more *uncertainties*, as she becomes able to suspend judgment on more statements. This phenomenon will not always allow for a suitable translation into a theory of conceptual changes. For instance, theories that rely on *frames* (such as Thagard 1992) have no clear way of dealing with it, insofar as it is not possible to suspend judgment on alternative conceptual frameworks from within a given frame system. Alternative proposals to represent conceptual changes (such as Kitcher's 1978) may take care of this difficulty, but I shall not pursue this possibility any further.

particular statement. In addition, “structural *weakening*” will refer to cases in which the agent ceases to have an attitude towards a statement she was aware of before; hence, after a structural weakening, the agent is no longer able to accept, reject, or suspend judgment about the statement.

§ 1.2. A Note on the Representation Apparatus

Before going any further, let me address a potential source of misunderstandings. At the time of studying theories of belief revision it is advisable to distinguish between at least three very different levels:

- (i) The level of the agent’s doubts and beliefs.
- (ii) The level of the agent’s attitudes (acceptance, rejection, or suspension of judgment) towards statements of her own language.
- (iii) The representation level.

Level (i) constitutes the agent’s *real* epistemic state.⁵ It should be clear that the agent’s doubts and beliefs, at level (i), are compatible with more than one linguistic manifestation of (at least part of) such states (at level (ii)), and, of course, with more than one modeling strategy. A well-known strategy (the one favored by AGM-style theories) involves the use of a formal language L , at level (iii), to represent acceptances, rejections, and suspensions of judgment of an agent at level (ii), which might be thought to reflect at least part of her doubts and beliefs, at level (i). In addition, L might be taken to be an idealization of a language the agent speaks; in this case, to say that the agent accepts that p (where “ p ” is a sentence of L) would amount to saying that she accepts a corresponding sentence in her own language (say, a suitable translation of p) – which in turn would

⁵ Let me recall here that most formal theories of belief revision do not distinguish between knowledge and belief – or between knowledge states and belief states. The theory offered in this paper will follow this trend. The rationale for this procedure can be found in a Peircean-based epistemology, according to which we need not justify prior beliefs in order to have *knowledge*; only *changes* are in need of justification. For contemporary elaborations on Peircean epistemology, cf. Fuhrmann (1997), ch. 1; Bilgrami (2000), (2004), or Levi (2004), ch.1, among others.

reflect a *bona fide* belief state. Notice that, in this scenario, the analytic tool would be compatible with many different approaches on the exact nature of the potential beliefs that are being so represented (sets of commitments, elements of a Boolean algebra, sets of dispositions, neurological events, etc.);⁶ in addition, there might be beliefs and doubts (at level (i)) that have no linguistic counterpart; our hypothetical theory would simply be silent about them.

Clearly, other ways of establishing relationships among the three levels are possible. For example, we might modify the example given above somewhat and claim that *L*, at level (iii), need not be an idealization of a language the agent speaks. We might also attempt to develop a representation theory that avoids the use of a linguistic device altogether. Alternatively, we might aim at a representation strategy that eliminates references to “beliefs” from the theoretician’s vocabulary – say, in case we were particularly interested in how agents should accept or reject sentences that might lack truth-values, while also endorsing the idea that the term “belief” commits us to talking about truth-valued items of some kind.

In this paper I shall develop a representation tool that requires a further distinction, in addition to (i), (ii) and (iii). Notice, first, that we can know all there is to know about which statements of her own language an agent accepts, rejects, or is in suspense, without knowing much about which such statements she takes to be true or false. To be sure, agents are unlikely to accept statements they take to be false, or reject statements they take to be true, but they might accept or reject statements that they take to be semantically undefined, for whatever reason. More generally, by merely looking at level (ii) we do not know whether the agent is, or is not, committed to bivalence. In short, a representation apparatus can convey all the information we deem relevant about level (ii), (and perhaps also about level (i), though this is naturally more contentious), without thereby conveying enough information about the agent’s semantic assumptions. Hence, in addition to levels (i), (ii) and (iii), consider also:

⁶ These options need not be pairwise incompatible, of course. Whether they are so or not depends on the way we understand each of these expressions.

- (ii') The level of the agent's semantic assumptions about statements of her own language.

For the most part, theories of belief revision have not been concerned with accurately reflecting (ii'), and the present proposal will not be an exception. However, I shall develop a modeling strategy in which a clear conceptual distinction between levels (i), (ii), (ii') and (iii) becomes crucial. As we shall see, the representation strategy that I favor relies on the construction of a semantics for a given regimented language. And, although at least part of such language might be thought of as an idealization of a language the agent actually speaks, the semantics that I shall use as a representation tool *is explicitly not to be confused with a semantics the agent actually endorses*. In general, the agent's semantic assumptions will not coincide with the semantic values yielded by the representation machinery, except for very special cases. It is clear, therefore, that the analysis will proceed from a third person point of view, as it were.

The last claim deserves some qualification. The semantics to be used as a representation tool should not be understood as the semantics the theoretician thinks that the agent should have, and it is not meant to convey the theoretician's own thoughts about the semantic values of the agent's language. Indeed, the goal will be to reconstruct *the agent's own attitudes and epistemic changes*, to the best of the theoretician's knowledge. Incidentally, for most cases the agent and the theoretician can turn out to be the very same physical individual (with one important exception to be duly mentioned), although they are always conceptually distinct.

I shall add further clarifications to the aforementioned distinctions in the course of my exposition of the model, and I shall come back to the philosophical import of my overall strategy in the last section.

§ 2. Modeling Structural Changes of Belief

My exposition of the model will be organized as follows. In section §2.1 I shall provide some definitions; section §2.2 will deal with structural strengthening and with what I

shall call “generalized expansions”; section §2.3 will deal with “generalized contractions,” and I shall address structural weakening in section § 2.4. Finally, in section §3 I shall offer some conclusions.

§ 2.1. Some Definitions

I shall assume that the complete epistemic state of an agent, at level (i), determines, at level (ii), a set of statements that the agent accepts, a set of statements that the agent rejects, and a set of statements about which the agent suspends judgment. I shall call them the agent’s “belief set,” her “set of disbeliefs,” and her “set of uncertainties,” respectively, and I shall say that beliefs, disbeliefs and uncertainties so captured by level (ii) constitute the *epistemically relevant background* of an agent, or, equivalently, her *propositional structure*.

I shall present my proposal from two different, but equivalent, perspectives. I shall call them “the semantic perspective” and “the axiomatic perspective” respectively.⁷ In the semantic approach I represent the complete epistemic state of an agent, as well as its possible modifications, by means of a model-theoretic account, and I also explain how to represent the idea that *a given epistemic state determines the agent’s belief set*. By way of contrast, in the axiomatic approach I represent the agent’s belief set and I give instructions as to how to transform one belief set into another (much in the way AGM originally proceeds); those instructions can typically be encoded in a bunch of axioms. The new belief set of an agent, as obtained by the axiomatic account after a belief change takes place, will be precisely the belief set that gets uniquely determined by the agent’s new total epistemic state, as obtained by the semantic account.

⁷ As it will become clear soon, the axiomatic perspective is not strictly speaking a *syntactic* approach, because it already presupposes a notion of semantic consequence.

§ 2.1.1. A Semantic Perspective

Let the epistemic state of an agent at a given time t be represented by a set \mathbf{M} of partial interpretations $I_1 \dots I_n$ for a propositional language L , *i.e.*, by a set of interpretations defined on subsets $\Sigma_1 \dots \Sigma_n$ of the set Σ of all well-formed formulas of L . Let us assume Σ to be defined in the usual way from a primitive vocabulary of atomic sentences $p_1, q_1, r_1 \dots p_n, q_n, r_n \dots$ by means of the connectives \neg, \vee (negation and disjunction), together with the usual punctuation symbols.

The motivation for the use of a set of partial interpretations is the following. We want to be able to represent beliefs, disbeliefs and uncertainties of an agent; at the same time, we also want to be able to refer to elements about which the agent has no epistemic attitude at all, because they are conceptually inaccessible for him. In the present proposal, beliefs, disbeliefs and uncertainties of the agent will be represented as sentences of L that come out *true, false* and *weakly undefined* in \mathbf{M} , respectively, in a way to be defined, whereas the conceptually inaccessible elements will be represented as sentences of L that end up being *strongly undefined* in \mathbf{M} . The fact that \mathbf{M} is a *set* of (partial) interpretations will help us obtain weakly undefined valuations, and the fact that at least some of the interpretations in \mathbf{M} are *partial* will help us obtain strongly undefined valuations. Let us spell out this basic idea in a more detailed fashion.

As we have said, \mathbf{M} is a set of partial interpretations $I_1 \dots I_n$ for L . For all $I_j \in \mathbf{M}$, we have $I_j: \Sigma_j \rightarrow \{\text{True}, \text{False}\}$, for some $\Sigma_j \subseteq \Sigma$. That is, each I_j is fully defined on a subset $\Sigma_j \subseteq \Sigma$. Let $\Sigma_{\mathbf{M}} = \bigcap \{\Sigma_j: I_j \in \mathbf{M}\}$. In other words, $\Sigma_{\mathbf{M}}$ is the largest subset of Σ such that each I_j of \mathbf{M} is fully defined on $\Sigma_{\mathbf{M}}$, for $j \leq n$. We will say that $\Sigma_{\mathbf{M}}$ represents the epistemically relevant background of the agent.

Note that, iff Σ_j is inductive, then defining I_j as a complete (standard) interpretation on Σ_j amounts effectively to construing I_j recursively from an assignment of truth-values I_j^* to a set of atoms in accordance with the truth-conditions of Bochvar's (or: so-called Kleene-weak) three-valued logic:

- If A is a propositional letter, A is True/ False in I_j iff $I_j^*(A) = \text{True}/ \text{False}$. Otherwise A is undefined in I_j .

- If $A = \neg B$, then A is True/ False in I_j iff $I_j(B) = \text{False/ True}$. Otherwise A is undefined in I_j .
- If $A = B \vee C$, then A is true in I_j iff (i) either $I_j(B)=\text{True}$ and $I_j(C)=\text{either True or False}$, or (ii) $I_j(C)=\text{True}$ and $I_j(B)=\text{either True or False}$. A is false iff $I_j(B)=I_j(C)=\text{False}$. Otherwise A is undefined.

If Σ_j is not inductive, then other strategies are possible, such as strong Kleene, or even a supervaluational semantics, both of which would have the effect of assigning a definite truth-value to some sentences that contain undefined components. However, in the following I shall confine myself to the mathematically easier case of an inductive Σ_j . As a result, the truth-value of a compound sentence is computed in the classical way when all components receive a truth-value, whereas the result is undefined whenever at least one of the components is undefined.

Next, we define:

- A is *True* in \mathbf{M} iff it is True in I_j , for all $I_j \in \mathbf{M}$;
- A is *False* in \mathbf{M} iff it is False in I_j , for all $I_j \in \mathbf{M}$;
- A is *undefined* in \mathbf{M} iff it is neither True nor False in \mathbf{M} . Then,
 - A is weakly *undefined* in \mathbf{M} iff it is undefined in \mathbf{M} and either True or False for all $I_j \in \mathbf{M}$.
 - A is *strongly undefined* in \mathbf{M} iff it is undefined in I_j , for some $I_j \in \mathbf{M}$ (hence A is also undefined in \mathbf{M})

As is obvious, we have used a sort of “supervaluational” account. Notice that $\Sigma_{\mathbf{M}}$ as defined above is precisely the set of sentences that are true, false or weakly undefined in \mathbf{M} , but not strongly undefined. Now, let $K_{\mathbf{M}} = \{A \in \Sigma : A \text{ is true in } \mathbf{M}\}$. We will say that the sentences in $K_{\mathbf{M}}$ represent the beliefs of the agent. And, as we have already mentioned, his disbeliefs are represented by the false sentences in \mathbf{M} , and his uncertainties by the weakly undefined sentences of Σ . In short, beliefs, disbeliefs and uncertainties are represented as sentences of $\Sigma_{\mathbf{M}}$, *i.e.*, as sentences of the epistemically relevant background. Thus, \mathbf{M} determines both an epistemically relevant background $\Sigma_{\mathbf{M}}$ and a belief set $K_{\mathbf{M}}$. Notice that $K_{\mathbf{M}}$ contains all classical tautologies of $\Sigma_{\mathbf{M}}$, but not of Σ .

K_M will not contain classical tautologies built out of propositional letters that remain undefined for some $I_j \in \mathbf{M}$.

Recall that, as I have already stated in section §1.2, I do not intend to claim that the agent *thinks* the sentences of L are actually true, false, or undefined: “true-in- \mathbf{M} ,” “false-in- \mathbf{M} ” and “undefined-in- \mathbf{M} ” are not to be understood as “true-for-the-agent,” “false-for-the-agent” and “without a truth-value for the agent.” By way of illustration, a sentence of L that comes out semantically undetermined in the model represents the fact that the agent is in suspense about certain sentence of her own language; the agent might (though she need not) think that the corresponding sentence (at level (ii)) does have a truth-value, of which she is ignorant.

§ 2.1.2. An Axiomatic Perspective

We will start by defining a consequence operator Cn on $\wp(\Sigma)$ [the power set of Σ], where Σ is as stated in §2.1.1. Let $X \subseteq \Sigma$ and let $A \in \Sigma$. Then, $A \in Cn(X)$ iff for all total interpretations I for Σ (defined in the standard, classical way), if $I(B)=\text{True}$ for all $B \in X$, then $I(A)=\text{True}$.

Given Σ , by Cn_S I shall mean the consequence operator defined on $\wp(\Sigma_S)$, for $\Sigma_S \subseteq \Sigma$.

From an axiomatic perspective, we will say that K is a *belief set* iff it is a set of sentences of Σ closed under the logical consequence operator Cn_S defined on $\wp(\Sigma_S)$, for some $\Sigma_S \subseteq \Sigma$, i.e., K is a belief set iff $K = Cn_S(K)$, for some $\Sigma_S \subseteq \Sigma$.

Now let me define the following epistemic concepts, according to how the agent stands towards an element A of Σ :

- The agent *accepts* A : $A \in K$
- The agent *rejects* A : $\neg A \in K$
- The agent is *in suspense* about A (or, in other words, A is *unknown* for the agent): $A \notin K, \neg A \notin K, A \vee \neg A \in K$

- The agent has *no attitude* about A (or, in other words, A is *epistemically inaccessible* for the agent): $A \vee \neg A \notin K$

It is easy to see that, if K_M is the set of sentences determined by a set M of partial interpretations for Σ , then K_M is a belief set in the axiomatic sense defined above: K_M is closed under the consequence operator Cn_M , *i.e.*, the consequence operator on Σ_M , where Σ_M is the epistemically relevant background of the agent. Conversely, if K is any logically closed set of sentences, let Σ_K be the fragment of Σ built inductively from the propositional letters occurring in the sentences in K . Then there will be at least some set of interpretations M_i that uniquely determines Σ_K and K . Actually, in the typical cases there will be a collection of such sets M_i ; the semantic account conveys more information than the axiomatic account, insofar as there are multiple ways in which a sentence can be strongly undefined in M . However, for any such M_i , there is a unique set of interpretations $M' \subseteq M_i$ fully defined for Σ_K and only for Σ_K that uniquely determines K . The extra information included in the semantic account is irrelevant as far as beliefs, disbeliefs and uncertainties of the agent are concerned.

§ 2.2. Structural Strengthening and Generalized Expansions

In a (non degenerate case of)⁸ *structural strengthening*, the complete epistemic state of an agent is modified in such a way that a former epistemically inaccessible sentence of Σ (as defined in §2.1.2) becomes a sentence the agent accepts or rejects, or a sentence about which she is in suspense – in other words, a former epistemically inaccessible sentence of Σ is turned into a sentence that belongs to the agent's propositional structure. I shall reserve the name of (non degenerate) *basic* structural strengthening for changes in which a former epistemically inaccessible sentence is turned into a sentence about which the

⁸ According to the terminology I favor, degenerate cases of epistemic changes actually involve no changes at all (*e.g.*, consider the “change” that consists in the addition of sentence A , and only A , to a belief set that already contains A).

agent is in suspense. In this case, the agent somehow becomes aware of a formerly inaccessible element, but ends up suspending judgment about it.

On the other hand, when a former epistemically inaccessible sentence of Σ becomes a sentence the agent accepts or rejects we shall say that a type of *expansion* has taken place, in addition to a structural strengthening. More generally, I shall talk of (non degenerate) *generalized expansions* to refer to shifts by which unknown or epistemically inaccessible sentences are turned into accepted or rejected. If a given sentence is unknown, but not epistemically inaccessible (*i.e.*, if it belongs to Σ_M), we will obtain a standard AGM expansion. As usual, if the sentence has already been accepted, then the complete epistemic state of the agent (and hence his belief set) should remain unchanged.

In short, we can have generalized expansions with and without (non degenerate) structural strengthening (depending on whether the sentence by which the agent expands was epistemically inaccessible or merely unknown), as well as (non degenerate) structural strengthening without expansion, in the technical sense just defined. We shall see in a moment, however, that any structural strengthening – even a basic structural strengthening – is bound to enlarge the belief set of the agent in various ways. In what follows I shall provide more precise definitions of the aforementioned concepts, both from a semantic and an axiomatic point of view.

§ 2.2.1. A Semantic Perspective for Basic Structural Strengthening and Generalized Expansions

Let M be an epistemic state, and let a be a *new* atomic sentence with respect to Σ_M (as determined by M). We define M_a as follows, for any non-empty M :

- (i) For each $I_j \in M$, there will be I_j' and I_j'' in M_a such that (1) $I_j'(a)=\text{True}$; (2) $I_j''(a)=\text{False}$; and (3) I_j' and I_j'' are equivalent to I_j on any other atomic sentence of Σ ;
- (ii) Nothing else is in M_a .

For the sake of completeness, we shall stipulate that, if $M = \emptyset$, then $M_a = \emptyset$.

Now, for $A \in \Sigma$, let $a_1 \dots a_n$ be the *new* atomic components in A (that is, new components with respect to Σ_M), in the alphabetic order. We define M_A simply as $(\dots((M_{a_1})_{a_2} \dots a_n))$. If A is already in Σ_M , then $n=0$ and $M_A = M$, of course. We will say that M_A constitutes a *basic structural strengthening* of M by A .

Next, let us define the epistemic state M *expanded by* A as $M+A = \{I_j \in M_A : I_j(A)=\text{True}\}$. If A was false in M , then $M+A = \emptyset$, as it should be.

As usual, $M+A$ determines a belief set $K+A = \{B \in \Sigma : B \text{ is True in } M+A\}$, and, analogously, M_A determines $\{B \in \Sigma : B \text{ is True in } M_A\}$,

Notice that if A is not strongly undefined, $M+A = \{I_j \in M : I_j(A)=\text{True}\}$, in which case $M+A \subseteq M$. And, in any event, regardless of whether A is, or is not, new to Σ_K , we will always obtain $K \subseteq K+A$. In the next section I shall consider further observations on expanded belief sets.

§ 2.2.2. An Axiomatic Perspective for Basic Structural Strengthening and Generalized Expansions

Let $\Sigma_S \subseteq \Sigma$ be a set of sentences as defined in §2.1.2, let $A \in \Sigma$, and let $\Sigma_{S,A}$ be the set of sentences built inductively from the atomic components occurring both in sentences of Σ_S and in A . Then $Cn_{S,A}$ is the consequence operator defined on $\wp(\Sigma_{S,A})$, as explained in §2.1.2.

Now let K be a belief set. As in §2.1.2, Σ_K is the fragment of Σ built inductively from the propositional letters occurring in the sentences in K . We shall say that $Cn_{K,A}(K)$ constitutes the *basic structural strengthening* of K by A . Clearly, when K is the belief set determined by M we obtain that $Cn_{K,A}(K) = \{B \in \Sigma : B \text{ is True in } M_A\}$, as desired. (Just notice that the set of all logically possible interpretations I_i fully defined for $\Sigma_{K,A}$ that make all members of K true coincide on $\Sigma_{K,A}$ with the set of all interpretations I_j in M_A .)

In addition, consider defining the *generalized expansion* of K with a sentence A , from the axiomatic perspective, as $\underline{K^{(+A)}} = Cn_{K,A}(K \cup \{A\})$. Then $\underline{K^{(+A)}}$ satisfies AGM

axioms for expansion, but replacing the unbounded consequence operator by $Cn_{K,A}$. It is also easy to see that, when K is the belief set determined by \mathbf{M} , $\underline{K^{(+A)}} = K+A$. (Just notice that the set of all logically possible interpretations I_i fully defined for $\Sigma_{K,A}$ that make both K and A true coincide on $\Sigma_{K,A}$ with the set of all interpretations I_j in $\mathbf{M}+A$.) If $\Sigma_{K,A}=\Sigma_K$, we are back at a standard, AGM expansion.

§ 2.3. Structural Strengthening and Generalized Contractions

The basic goal of a contraction is to turn the element to be contracted into an unknown element for the agent. That is, a contraction by A should have the effect that the agent ends up in a state of suspension of judgment about A – provided A is the type of sentence about which the agent can suspend judgment in the first place. More precisely, I shall define a *generalized contraction* by A (for $A \in \Sigma$) as the shift by which A becomes unknown for the agent (provided suspension of judgment is possible) regardless of its previous status. Thus, A need not have been in Σ_M prior to the contraction. We shall see that, if A was not in Σ_M , instructions to contract will be equivalent to performing a prior structural strengthening, followed by a standard, AGM-like contraction. For example, if A is a classical tautology but was not a former sentence of Σ_M , a generalized contraction by A should actually add A to the agent’s belief set. In general, generalized contractions might yield a final belief set that is neither a subset nor a superset of the original belief set.

As usual, we will start by stating the semantic point of view (§2.3.1), and then will proceed to give a possible axiomatization from a belief-set perspective (§2.3.2).

There is another type of contracting mechanism that may draw our attention at this point, namely, the case in which certain element A becomes epistemically inaccessible for the agent – so that her epistemically relevant background gets weaker. I shall refer to this case as a process of “forgetting”, or structural weakening, and not just a contraction. I deal with it separately (and very briefly) in section § 2.4.

§ 2.3.1. A Semantic Perspective for Generalized Contractions

Let us start by defining a **Contraction Set** C^M_A (*i.e.*, a contraction set C_A relative to M) as follows. Let a be any atomic sentence of Σ , not necessarily new to Σ_M . For $M \neq \emptyset$, C^M_a will be such that,

- (i) for each $I_j \in M$, there will be I_j' and I_j'' in C^M_a such that (1) $I_j'(a)=\text{True}$; (2) $I_j''(a)=\text{False}$; and (3) I_j' and I_j'' are equivalent to I_j on any other atomic sentence of Σ ;
- (ii) nothing else is in C^M_a .

As before, for the sake of completeness: if $M = \emptyset$, then $C^M_a = \emptyset$

Next, if A is any sentence of Σ , let $a_1 \dots a_n$ be the atomic components in A , in the alphabetic order. We define C^M_A simply as $(\dots((C^M_{a1})_{a2} \dots a_n))$. In what follows I will omit the superscript when the reference to M is clear enough.

Notice that $M_A \subseteq C_A$ (for M_A as defined in section §2.2.1). Also, if A is already in Σ_M , then $M \subseteq C_A$; in addition, if A is true in M , then $M \subset C_A$. On the other hand, if *all* atomic components in A are new with respect to Σ_M , C_A is just M_A .

Consider now a strongly connected and transitive preference relation \succsim_C among interpretations of a given contraction set C , such that:

- (a) For any I_j and I_j' in C_A : If $I_j \succsim_{C_A} I_j'$, then either (1) $I_j \in M_A$, or (2) $I_j(A)=\text{False}$ and for any other I_i in M_A , $I_i(A)=\text{True}$.
- (b) If $A \leftrightarrow B \in Cn_{A,B}(\emptyset)$, then I_j is a preferred interpretation of C^{MB}_A , [according to \succsim defined for C_A relative to M_B] iff I_j is a preferred interpretation of C^{MA}_B [according to \succsim defined for C_B relative to M_A]

I shall omit the subscript in “ \succsim ” when the contraction set under consideration is apparent from the context. It is clear that (a) and (b) do not determine a unique preference relation for a given contraction set. This is as it should be, and it is meant to reflect the fact that not all admissible belief change strategies are equally worthy, as far as the agent is

concerned. Of course, agents should be allowed to modify their belief change preferences across time, which should be duly reflected, at the theoretician's level, as changes in \succsim . But in this paper I shall not be concerned with this particular type of shift.

We define the **generalized contraction** of M by A as the epistemic state $M-A = \{I_j \in C_A: I_j \succsim I_j', \text{ for any other } I_j' \text{ in } C_A\}$. As usual, $M-A$ determines a belief set $K-A = \{B \in \Sigma: B \text{ is true in } M-A\}$.

Some consequences of the present definition are worth considering. First, notice that if A is already in Σ_M , then $M \subseteq M-A$. In addition, if all atomic components of A are new for Σ_M , then $M-A = C_A$. Finally, when A is an epistemically inaccessible sentence (*i.e.*, a sentence containing at least some atomic components that extends Σ_M) we actually *expand* the belief set of the agent with new sentences. In particular, if A is a classical tautology, we will actually add it. For instance, let A be $p \vee q$, where q is not in Σ_M . In this case, we will end up having $q \vee \neg q$ in $K-(p \vee q)$, as well as $q \vee B$, for any sentence B of K_M that remains in the final belief set. Also, if p was in K_M , $K-(p \vee q)$ will contain $p \vee \neg q$, but not p (by restriction (a) on \succsim). So the resulting belief set will be neither a subset nor a superset of the original one. As we shall see in the next section, the result is equivalent to performing a standard AGM contraction on $Cn_{K,A}(K)$, rather than on K .

§ 2.3.2. An Axiomatic Perspective for Generalized Contraction

Let $\underline{K}^{(-A)}$ be a belief set obtained from K that satisfies the following postulates:

- (-1) $\underline{K}^{(-A)} = Cn_{K,A}(\underline{K}^{(-A)})$ (closure)
- (-2) $\underline{K}^{(-A)} \subseteq Cn_{K,A}(K)$ (inclusion)
- (-3) If $A \notin Cn_{K,A}(K)$, then $\underline{K}^{(-A)} = Cn_{K,A}(K)$ (vacuity)
- (-4) If $A \notin Cn_{K,A}(\emptyset)$, then $A \notin \underline{K}^{(-A)}$ (success)
- (-5) $Cn_{K,A}(K) \subseteq Cn_{K,A}(\underline{K}^{(-A)} \cup \{A\})$ (recovery)
- (-6) If $A \leftrightarrow B \in Cn_{A,B}(\emptyset)$, then $[Cn_{K,A,B}(K)]^{(-A)} = [Cn_{K,A,B}(K)]^{(-B)}$ (preservation)

These postulates are identical to AGM's when Cn is unbounded. Hence, we can profit from AGM's well-known results and replace “ K ” by “ $Cn_{K,A}(K)$ ” to obtain $\underline{K^{(-A)}} = \cap\gamma(Cn_{K,A}(K)\perp A)$, for “ \perp ” and “ γ ” as presented in section §1.1.

It is easy to see that, for a given selection function γ over the members of $Cn_{K,A}(K)\perp A$, there is some preference relation \succsim over the interpretations of C_A that yields $K-A$.

Proof: Consider the collection of sets $Q_1\dots Q_n$, such that, for each I_i in C_A that yields A false, $Q_i = \{B \in \Sigma: B \text{ is true in } M_A \cup \{I_j\}\}$. Each Q_i is a maximally consistent subset of $Cn_{K,A}(K)$ that does not imply A . Hence $\{Q_1\dots Q_n\} = (Cn_{K,A}(K)\perp A)$, and, for any well-defined γ , $\cap\gamma\{Q_1\dots Q_n\} = \cap\gamma(Cn_{K,A}(K)\perp A)$. Now suppose that $D \in \cap\gamma\{Q_1\dots Q_n\}$. Then D is true in $M_A \cup \{I_1\dots I_j\}$, for a selected $I_1\dots I_j$ of C_A that make A false. Let \succsim be the preference relation defined for C_A that yields exactly $M_A \cup \{I_1\dots I_j\}$ as preferred interpretations, and we are done. ■

Hence, any set $\underline{K^{(-A)}}$ that fulfills axioms (–1) to (–6) is included in $K-A$, as determined by an appropriate \succsim . The converse is straightforward as well: we obtain that all instances of (–1) to (–6) hold for $K-A$. Let me refer to those instances as (–1S) to (–6S), respectively – where the “S” stands for “semantic”:

Proof: (–1S) Left to right is trivial. For right to left, assume sentence D is in $Cn_{K,A}(K-A)$; hence all I_j fully defined for $\Sigma_{K,A}$ that make all members of $K-A$ true make D true as well. Hence there cannot be any I_i in $M-A$ that makes D false. Thus we obtain $K-A = Cn_{K,A}(K-A)$.

(–2S) Assume D is in $K-A$. Hence all I_j in $M-A$ make D true. As $M_A \subseteq M-A$, there is no I_i in M_A that makes D false. Hence D is in $Cn_{K,A}(K)$.

(–3S) Suppose there is some I_j fully defined for $\Sigma_{K,A}$ that makes all members of K true and A false. By construction, there is some I_i in M_A that coincides with I_j on

$\Sigma_{K,A}$. Hence, by restriction (a) on \succcurlyeq , $\mathbf{M}\text{-}A = \mathbf{M}_A$. Hence $K\text{-}A = Cn_{K,A}(K)$, as desired.

(–4S) Suppose there is some I_j fully defined for $\Sigma_{K,A}$ that makes A false. By construction, there is some I_i in C_A that coincides with I_j on $\Sigma_{K,A}$, and hence there is at least some interpretation in $\mathbf{M}\text{-}A$ that makes A false. Hence, $A \notin K\text{-}A$.

(–5S) Suppose $D \notin Cn_{K,A}(K\text{-}A \cup \{A\})$. Then there is some I_j fully defined for $\Sigma_{K,A}$ that makes all members of $K\text{-}A$ true, A true, and D false. Suppose there is some I_i in $C_A \setminus \mathbf{M}_A$ that coincides with I_j on $\Sigma_{K,A}$. As $I_j(A)=\text{True}$, $I_i \notin \mathbf{M}\text{-}A$, so it does not make all members of $K\text{-}A$ true, contrary to our assumption. So $I_i \in \mathbf{M}_A$. Hence $D \notin Cn_{K,A}(K)$, and we obtain $Cn_{K,A}(K) \subseteq Cn_{K,A}((K\text{-}A \cup \{A\}))$, as desired.

(–6S) This is straightforward from restriction (b) on \succcurlyeq . Just notice that $[Cn_{K,A,B}(K)]\text{-}A = \{D \in \Sigma: D \text{ is true in } \mathbf{M}_{B\text{-}A}\} = \{D \in \Sigma: D \text{ is true in the set of all preferred interpretations of } \mathbf{C}^{MB}_A\}$, whereas $[Cn_{K,A,B}(K)]\text{-}B = \{D \in \Sigma: D \text{ is true in } \mathbf{M}_{A\text{-}B}\} = \{D \in \Sigma: D \text{ is true in the set of all preferred interpretations of } \mathbf{C}^{MA}_B\}$. Hence, if $A \leftrightarrow B \in Cn_{A,B}(\emptyset)$, then $[Cn_{K,A,B}(K)]\text{-}A = [Cn_{K,A,B}(K)]\text{-}B$. ■

Thus, $K\text{-}A = \underline{K^{(-A)}}$, and we obtain an AGM contraction as a limiting case when $\Sigma = \Sigma_{K,A}$. Incidentally, if $\Sigma = \Sigma_{K,A}$ and \succcurlyeq [defined for C_A] picks out at most one additional I_j from $C_A \setminus \mathbf{M}_A$, we obtain a maxichoice contraction, and if $\Sigma = \Sigma_{K,A}$ and \succcurlyeq picks out all admissible I_j from $C_A \setminus \mathbf{M}_A$, we achieve a full meet contraction.

Before moving to the next section, let me say a few words on the preservation postulate. Notice, first, that we could have replaced “ $\underline{K^{(-A)}}$ ” by “ $[Cn_{K,A}(K)]^{(-A)}$ ” all along (–1) to (–5), and, correspondingly, we could have replaced “ $K\text{-}A$ ” by “ $[Cn_{K,A}(K)]\text{-}A$ ” in (–1S) to (–5S): it is easy to see that \mathbf{C}^M_A [that is, C_A relative to \mathbf{M}] = \mathbf{C}^{MA}_A [C_A relative to \mathbf{M}_A], hence $\mathbf{M}_{A\text{-}A} = \mathbf{M}\text{-}A$, and $[Cn_{K,A}(K)]\text{-}A = K\text{-}A$. By way of contrast, $[Cn_{K,A,B}(K)]^{(-A)}$ need not coincide with $Cn_{K,A,B}(\underline{K^{(-A)}})$, and hence postulate (–6) as stated above is not equivalent to

$$(–6') \text{ If } A \leftrightarrow B \in Cn_{A,B}(\emptyset), \text{ then } Cn_{K,A,B}(\underline{K^{(-A)}}) = Cn_{K,A,B}(\underline{K^{(-B)}})$$

The correct translation of AGM preservation postulate to a setting with bounded consequence operators is (–6), and not (–6’). To see this, notice that we cannot assume $Cn_{K,A,B}(\cap\gamma(Cn_{K,A}(K)\perp A))$ to be identical with $Cn_{K,A,B}(\cap\gamma(Cn_{K,B}(K)\perp B))$, as in each case the selection function is defined for different sets. Likewise, $M_{AB}\text{--}A = M_{B\text{--}A} \neq (M\text{--}A)_B$, unless $B \in \Sigma_{K,A}$. Hence,

$$(-6'S) \text{ If } A \leftrightarrow B \in Cn_{A,B}(\emptyset), \text{ then } Cn_{K,A,B}(K\text{--}A) = Cn_{K,A,B}(K\text{--}B)$$

does not hold in general when $C^M_A \neq C^M_B$. For a counterexample, just let $K=Cn_p(\{p\})$; $A=p\vee\neg(q\vee\neg q)$; and $B=p\vee\neg(r\vee\neg r)$. Then, unless \succ_{CA} [i.e., \succ defined for C_A] is such that $M\text{--}A=C_A$, and \succ_{CB} [i.e., \succ defined for C_B] is such that $M\text{--}B=C_B$, $(M\text{--}A)_B$ will not coincide with $(M\text{--}B)_A$, and hence we will have $Cn_{K,A,B}(K\text{--}A) \neq Cn_{K,A,B}(K\text{--}B)$.

§ 2.4. Structural Weakening: The Case for “Forgetting”

So far, we have not considered cases in which the propositional structure of an agent is weakened, that is, cases in which an agent ends up having a new epistemically relevant background that is a proper subset of her old one. In order to turn a (possibly accepted) sentence A into an epistemically inaccessible one, consider the following sequence of events:

- Contract the belief set K by A .
- Identify a finite base X of $K\text{--}A$, such that no atomic component of A appears in a sentence of X , and such that $Cn_K(X) = K\text{--}A$.
- Make the closure $Cn_{K\text{--}A}(X)$, for $Cn_{K\text{--}A}$ defined on $\wp(\Sigma_{K\text{--}A}) \subseteq \wp(\Sigma_K)$.

This rational reconstruction of a process of “forgetting” may not be implemented if we are unable to find a base X as indicated, for whatever reason. In addition, notice that in a structural weakening the theoretician is not only conceptually distinct from the agent, but also *physically* distinct, as the theoretician is trying to capture the fact that the agent no longer recognizes certain statements as statements of her own language.

§ 3. Conclusions

Let me finish by offering a general assessment of the belief change model that I have presented here. Notice, first, that – as promised – it extends standard theories of belief revision, in the sense that it provides precise instructions to represent structural shifts of belief. And, as it happens, allowing a stronger sense of ignorance to enter the picture yields a richer ideal of rationality: as opposed to standard theories, in the present model agents can no longer be maximally opinionated in absolute terms, but with respect to a certain background (what I have called here the “epistemically relevant background”) that is itself subject to changes. Thus, the present model helps us remember that the aim of inquiry cannot be that of securing a “true and complete story of the world,” insofar as any such theory is bound to be provisory, and inserted in a larger context.

A model for structural strengthening can find a natural application in rational reconstructions of scientific discoveries. More generally, it can be included in a more comprehensive account of the concept of inference to the best explanation.⁹ On the other hand, it has often been observed that scientists are sometimes just ignorant of theories endorsed by former scientific communities. This can be naturally modeled, from the theoretician’s point of view, as a structural weakening, as proposed in this paper.

I would like to add a final comment on the chosen formal strategy. Someone might complain that it is philosophically unwise to use a formal apparatus that relies on a semantic valuation of sentences which is explicitly alien to the agent’s own semantics. I disagree. To address this objection, consider first what the theoretician actually does. Notice that the theoretician is instructed to pay attention to the agent’s attitudes (does the agent accept, reject, or suspend judgment on certain statements? does she ignore the very

⁹ Typically, a process of inference to the best explanation consists of an abductive stage, in which a set of options is enlarged with new elements, and a selection stage, in which the agent chooses a preferred element from the prior set of options. The proposal offered in this paper could be used to model the abductive stage, and we can seek to supplement it with an apparatus to assign preferences between rival explanatory hypotheses, to account for the process of choosing the most explanatory hypotheses from the lot. For a more detailed description of how to model a full-fledged IBE process, see my (2006).

possibility of accepting, rejecting, or suspending judgment on others?), and she is also instructed to correlate those attitudes with certain specific attribution of semantic values to sentences of a particular formal language. As a result of this, the theoretician makes a correlation between what the agent takes to know and ignore, and *a full description of a possible world* – namely, the world that makes true the sentences of L that are true in M , false the sentences of L that are false in M , and weakly and strongly undefined the sentences of L that are weakly and strongly undefined in M , respectively.¹⁰ More precisely, we can say that the theoretician is instructed to reconstruct how the world would be if the sentences in Σ_M were adequate translations of a language the agent spoke, and the agent's epistemic state (at a given time t) were *definitive*, in the sense that there were nothing else to learn, objectively speaking.

Of course, sane agents do not take the actual world to be identical to the possible world that gets so defined by their ignorance. Sane agents do not think of themselves as the source of semantics, as it were – so it is no surprise that the semantics of the model is not their own.¹¹ Still, the overall picture that results from the present proposal makes for a nice visual effect, I think: different knowledge states can be correlated with points in a map of possible worlds, where each world completely determines the semantic values (including, possibly, two types of semantic indeterminacy) of sentences of a given representation language. More generally, we obtain that a path of epistemic changes draws a particular pattern in the aforementioned map. Insofar as we are careful enough so

¹⁰ As is obvious, worlds that are so correlated to epistemic states should make room for semantic indeterminacy. For the present purposes, it is not necessary to take a stance on what features of such worlds are those that end up being captured by weakly and strongly undefined sentences.

¹¹ Just to clarify possible misunderstandings, let me consider here under which circumstances the agent's semantic values and those embedded in the representation tool do indeed agree. Assuming L includes as a proper part a language the agent speaks, the two valuations coincide for the epistemically relevant bit of L when the agent is opinionated and holds a bivalent semantics. If the agent is opinionated but not committed to bivalence, the two valuations might differ, because acceptances and rejections with no truth-value for the agent will be rendered as true in M and false in M , respectively. On the other hand, assume that the agent is opinionated and that she accepts and rejects statements if and only if she thinks they have a truth-value, whereas she suspends judgment on statements with no truth-value. Then again the two valuations might (but need not) coincide for the epistemically relevant bit of L , depending on whether the agent accepts, or does not accept, tautologies made out of elements that lack a truth-value from her point of view.

as not to take the representation tool for the real thing, this strategy conveys a graphic metaphor of the way knowledge evolves.

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